

The background of the page features a large, light gray watermark of the University of Chicago crest. The crest is a shield divided into two horizontal sections. The top section has a background of vertical lines and contains an open book with the Latin motto "Artes Liberales CatSci Exco-entia latur" written on it. The bottom section contains a bald eagle with its wings spread, perched on a laurel wreath.

On the Objectivity of Logic Choice

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In mathematical and philosophical inquiry, the field of logic has long held special status. Logic, conceived generically as the study of valid reasoning and as the arbitrator of true statements, has been seen as immune to questions of its own justification or objectivity. This immunity has been attributed to some obvious or constitutive property of logic that has made questions on these dimensions seem confused or misplaced.

However, as the study of logic has formalized and been analyzed in detail, certain potentially problematic features have become more apparent. The codification and elaboration of the field of logic has shown that it is not the study of a monolithic, singular object. Instead, there has been the delineation of many possible orders of logic, different modes of inference, and divergent interpretations of the logical connectives. Each of these systems of logic comes along with many possible philosophical choices and commitments. Each system yields varying systematic consequences; differing systems may or may not be conservative with respect to each other, individually complete, or even mutually comparable in a meaningful way.

It is evident that one logic may be more applicable than another to certain situations: second-order logic is the clear simple choice for induction problems, relevant logics for handling context dependence, or first-order logic for most mathematical theories. This underscores the question of which system, which extension or formulation in general, is the *correct* one? This question – call it the Choice – will be our focus. This question additionally motivates other interconnected and relevant questions that we will also consider. Can the Choice be independently justified

and warranted? Are questions of objectivity even relevant or meaningful for the Choice? What implications for and commitments to views regarding truth does the Choice entail? Agreement on what constitutes a logically valid argument or proof is central to the projects of mathematicians and philosophers alike and as such demands close scrutiny. Answers to the above questions are needed to ensure that continued discourse on logic choice can amount to more than differing thinkers talking past one another.

1 What is Logic? What is a logic?

In order to speak about the field of rival logics and possible interplay between them, it makes sense to first provide a generic definition of what can or should qualify as a logic in the following discussion. Under its initial formalizations, logic was seen, as W.V.O. Quine put it in the introduction to his book *The Philosophy of Logic*, as “the systematic study of the logical truths” (vii). This seemingly attractive conception locates logic as the practice of formalizing logical truth, understanding its reach, and arbitrating its tautologies. However, this truth-based conception has been shown to be unwieldy as it necessitates a fully-fledged philosophy of truth within which a study of logical truth may be conducted.

In general, logic can instead be more simply and accessibly thought of as not being about *truth*, but instead about *consequence*. With consequence taking primacy, logic then is the tool by which inference is formalized, validity in argumentation can be verified, and a full explanation of the relationships between premise and conclusion made clear. When conceived in this consequence-grounded way, a

particularly flexible, substantive, and generalized definition for what could count as a logic becomes available. Rather than having to clarify complex philosophical points regarding the nature of truth, a view of logic that regards consequence as the primary relation allows a definition of logic simply based on sets of premises and conclusions and nothing more.¹

The above requirements describing formalization, validity, and consequence relation are met when rules of inference between sentences in a formal (usually interpreted) language are codified. Under these criteria, all of the familiar developed logics can be easily included in the discussion, including constructivist, first-order, second-order, and modal logics. Constructive, first-order, and second-order will be sufficient rival logics in order to explore the space of objectivity in the Choice that is our topic. These three are sufficient because between them, there are examples of the two main types of disputes between rival logics. Both *status disputes* about particular sentences and *extension disputes* over which sentences are truth-apt (capable of accepting a truth condition) become apparent when considering these different logical systems.

Due to the elucidation of many rival logics that are available to logician and philosopher alike, there is an initial impulse to want to discuss similarities and

¹The move from a truth-centered to a consequence-centered conception for logic allows for a structurally streamlined and simplified treatment of the subject. On a first glance, this change in focus seems to avoid the messy philosophical details of having to deal with robustly defining and describing “logical truth” specifically, or “truth” more generally. However, the consequence-based view doesn’t entirely escape this problem. Relevant to later discussion, it should be noted that an argument from premises a_1, a_2, \dots, a_n to c is valid only whenever the sentence conjoining all of the premises into a conditional with the conclusion is logically true. Additionally, one can reduce logical truth to a degenerate case of logical consequence when the assumption set is empty. Despite the desire to disassociate logic from truth by focusing on consequence, the interdependence is unavoidable.

differences between these systems. Indeed, this impulse seems to extend to a desire to evaluate the merits of rival logics. Intuitions of this kind, however, presuppose that there is a single system in which such comparisons can be meaningfully made. Attempting a full analytical explanation and defense of these informal ideas has proven to be difficult at best, despite the apparent value of such foundational work.

The question of which logic to use in everyday practice, as well as formal philosophical or mathematical practice, is one with vital importance and far-reaching consequences. In particular, evaluating arguments for validity and consistency is central to philosophical practice. Often, arguments are presented with an assumption that the inferences, connectives, and quantification used are universally understood, and therefore not part of the debate. These presuppositions need to be subjected to close critical analysis. First, note that such assumptions smuggle in strong claims about the nature and existence of a priori knowledge. The assumption of a singular universal object logic within which a conversation occurs directly entails that certain statements are not only true, but are necessary, a priori true. These entailed statements, the logical truths, therefore import wholesale an entire set of statements regarding a priori knowledge.

Additionally, when discussing the objective nature of the Choice, it is important to distinguish between discussions in a particular object language and discussions in the meta-language. Confusion on this point can lead to many non-starting arguments, as a 'proof' of objectivity within a particular object language can hold little to no weight in discussions intended for the meta-language. However, properly drawing the line between object- and meta-language when dealing with logic as an

object of philosophical investigation can be difficult and complicated, due to the common practice of conflating the two languages.

Particular choices in logic, such as the choice between arguments A or B in attempting entailment for conclusion c , are choices within a particular object language. It is a commitment to what rules we will follow in a formal sense when discussing objects of a non-logical nature. When speaking *about* logics and logical thinking, however, we move into the meta-logical arena. Here is where the difficulties of selecting what constitutes a valid meta-logical argument become apparent. For logic choice to be considered part of an objective discourse, the arguments for or against a particular object language should be considered objective at the meta-logical level. But what choice of logic should be used to evaluate these meta-logical statements? Although we will detail many possible answers to this question below, there does seem to be a potentially natural suggestion. In the vast majority of logical presentations, the metalogic used is classical first-order logic. If we are always using classical logic in our top-down analysis, can we conclude that classical logic is in fact just the right logic? At this point in our investigation we lack the theoretical framework to provide a clear answer, but we will return to this idea to show at the end that such a conclusion cannot be strongly supported.

For the solutions investigated in detail below, we will compare each story's ability to satisfy all four of the following desired properties: objectivity, consistency, normativity, and epistemic accessibility. Outside the scope of this paper will be considering logics that allow for inconsistency or epistemic inaccessibility; only solutions that are known to satisfy these properties will be investigated. For the

dimension of normativity, we will try to hold constant a notion that a logic should be normative in the sense that all valid arguments should be able to be *judged* by a removed party. Arguments then can be seen as correct or not in a “looking over a logician’s shoulder” sense; it would be a *mistake* to assert the premises of a valid argument while denying its conclusions. The following investigation will hold fixed the criteria of consistency, accessibility, and normativity in order to see the effect of different solutions to the satisfaction of the objective criterion. Between the following solutions, there seems to possibly be a trading game between desired components. The four properties under investigation seem to have an air of mutual exclusivity. The question of objectivity of logic choice seems to go right to the heart of the larger meta-logical issue: do these dimensions capture mutually independent properties that are simultaneously satisfiable? If a positive answer can be found for the question posed by the Choice, then a positive answer for this larger question has hope of solution.

2 The Space for Answers Regarding Logic Choice

To help get an understanding of the landscape, let us lay out three broad territories in which an answer to the Choice might be found. A first category of response to the Choice is that this kind of question is irrelevant because logic choice is not apt for such analysis. This position asserts that there is a primacy to logic choice over notions of objectivity. Namely, it assumes that to have a viable framework with which to discuss problems of objectivity, a system of inference and truth – and therefore logic – must already be in place. By putting logic first, and thus above

discourses of objectivity, the question of whether a logic is correct or incorrect loses significant meaning. However, this initial response to the central question seems unsatisfactory. In practice, philosophers seem more than capable of understanding the qualities and attributes of the rival logics and comparing them to one another. The apparent ease of this very act suggests that these objects are apt for comparison on at least some level and therefore that questions of objectivity in choice are not wholly devoid of relevance. However, this may be a unfounded intuition.

Another potential route to an answer seems to lie in the assumption that a unified adjudicating logic can be located in the meta-logical discussion. In order to salvage a notion of objectivity without resorting to accepting certain circularities, the assumption of a knowable, useable über-logic would allow for the successful evaluation of these object language disputes. However, there are large questions about what form this über-logic would take and how it would operate. Can it be formalized like the object languages, or must such considerations only be processed within a system of informal reasoning? If such formalization were not possible, then we would be compelled by the considerations of the first section to reject it as a legitimate logic. Alternatively, would it resemble or replicate one of the formal logics themselves, or perhaps a natural naive language which admits certain paradoxical statements? Any formal system strong enough to encompass all the relevant object logics seems capable of extending beyond what is conventionally considered to be “knowable” in a finitistic or even rule motivated sort of way.

The last route to an answer seems to be the one suggested by the arguments of Paul Boghossian (2000) for the possibility of objective epistemic reasons: a particular

logic can find objective justification, but only relative to itself. Similar in form to the first suggested resolution, this solution rejects the possibility of a grand unified meta-logic which can meaningfully adjudicate the differences between the rival logics. This solution still asserts that an objective justification is possible through the use of a rule- or functional-circularity. In this way, certain justificatory arguments can bootstrap themselves, recursively using their own rules in the process. Boghossian argues that this argument can succeed without introducing vicious circularities or contradictions. This tactic seems like an inviting middle ground by attempting to dissuade epistemic relativism while recognizing the structural difficulties that exist within any meta-logical discussion. However, the solution requires the admission of a particular form of argument that seems troubling due to its circularity. In potentially solving one problem, are we inviting new ones to the discussion? Arguably, this solution still only provides relativized objectivity, when considering that multiple differing logics can be meaningfully justified through the bootstrapping procedure.

In order to satisfy our requirements of epistemic accessibility and consistency, answers to the Choice which suggest that it is meaningless or not apt for close philosophical analysis seem to be non-starters. By construction, they preclude the possibility of knowing an answer to the Choice and violate our accessibility criterion. Accordingly, we will only consider solutions pursued from the last two routes. Considered first will be the story of objectivity offered by Crispin Wright in *Truth and Objectivity*. This story can be seen as a standard and traditional story of objectivity within a singular logic which is unified across the object- and meta-

logical level, as framed within long standing realist/anti-realist debates. Second, we will consider Boghossian's rule-circular suggestion pursued within his (2001). After framing these arguments within the context of the Choice, we will judge their success at unveiling a substantive dimension of objectivity while respecting our normative constraint.

3 Robust Objectivity of Choice

As a first investigation of the objectivity of logic choice, we will explore whether specially developed criteria for objectivity are satisfied. Crispin Wright's *Truth and Objectivity* (1992) is a thorough study into ideas of objectivity. In attempting to clarify the terms of long-standing realist/anti-realist debates across diverse philosophical disciplines, Wright develops a small set of evaluative tools for judging the objective nature of a given discourse, or its aptitude for realist construal.²

Wright's tests for objectivity are based on the idea of a minimalist conception of truth and truth-aptitude. For Wright, a minimalist view towards truth can be summarized as the position that for "a predicate to qualify as a truth predicate, [it must] satisfy each of a basic set of platitudes about truth: the platitudes, for instance, that to assert a statement is to present it as true; that 'S' is true if and only if S; that statements which are apt for truth have negations which are likewise..." (Wright, 1996, 864). Additionally, a minimalist view towards truth-aptitude is one that asserts that any discourse's propositions that evince certain syntactic features

²Although the concepts of objectivity and realist construal do not generally align, on Wright's minimalist conception of truth and truth-aptitude these concepts are coextensive.

and are disciplined can have truth conditions meaningfully applied to them. Under this minimalist view of truth and truth-aptness, it seems eminently reasonable to consider discussion of logic choice to be subject to a construal regarding minimal truth: statements regarding the correctness or incorrectness of a choice in logic have the structural requirements desired.³

3.1 Epistemic Constraint

The first test developed by Wright as a criterion for objectivity is that of Epistemic Constraint. A discourse is considered to be epistemically constrained if it is not possible for there to be unknowable truths in the discourse. In particular, a discourse can be considered constrained if it is possible to rule out unknowable truths on an a priori basis. According to Wright, if a discourse is epistemically unconstrained, then any subjective or irrealist explanation of that discourse fails to get off the ground. Because of this, Epistemic Constraint forms a strong first test of objectivity: unconstrained discourse can only be successfully accounted for on a realist interpretation, allowing for the construction and supposition of objective justifications.

Asserting that a particular meta-logical statement regarding the Choice is true can be equivalently understood as the asserting of a justification for a particular epistemic principle. For instance, asserting that first-order logic is the *objectively correct* logic is equivalent to claiming the objective truth of an epistemic principle which guarantees the validity of first-order arguments and truth of first-order conclusions. It is worth remembering at this point that the discourse regarding

³For a more detailed specification of Wright's conceptions of minimalism, see the first two chapters of Wright (1992)

choice between rival logics must exist at the meta-level, and not object-level. So, how epistemically constrained is the Choice? In order to answer this question, let us first assume that the Choice is epistemically unconstrained. This assumption would imply that there would exist true meta-logical statements regarding rival logics that are not knowable under any circumstances. This situation has the air of distinct implausibility – it is supposed that we can know, or at least assert, the existence of certain epistemic principles, like the acceptability of reasoning in accordance with *modus ponens* (MP). These principles would invariably be used in answering the Choice. However, if the Choice were a member of an unconstrained epistemic discourse, there would be true, yet unknowable, inaccessible epistemic principles. This leaves the unpalatable condition of assertable but unknowable justifications for epistemic principles used to answer the Choice.

This initial sense of implausibility is shared by Paul Boghossian in his investigations regarding objective epistemic reasons in (2001), investigations we will engage in detail later in this paper. On this point Boghossian makes a strong argument *against* the discourse being epistemically unconstrained:

It would be peculiar, to say the least, if truths of this type were in principle unknowable. It would certainly be peculiar for us to *suppose* that they are unknowable. For in what could our confidence that there are such facts consist, if we simultaneously take it that we cannot know what they are? Prima facie, indeed, a much stronger claim seems plausible: not merely that these facts are knowable, but that they are, at least in large measure *known*. For are we seriously to suppose that we don't know what it takes to justify a belief or a claim? (Boghossian, 2001, 6)

If we follow Boghossian's line of thought, there will emerge a serious question as to the possibility of a discourse surrounding the Choice which is epistemically

unconstrained. We are left to conclude that on Wright's most definitive dimension of objectivity, no informative answer to the question of the objectivity of logic choice obtains. If the discourse could be seen as unconstrained, a realist construal would provide a strongly objective justification for any answer to the Choice. However, Boghossian's comments make it clear that this is not the case; the discourse seems fundamentally and constitutively epistemically constrained. Therefore, this test fails to inform substantially, and the discourse regarding the Choice could successfully manage a realist or anti-realist construal.

After Epistemic Constraint, Wright develops three other tests which are more heuristic in their identification of an objective discourse. These are entitled Cognitive Command, Width of Cosmological Role, and Euthyphro Contrast. We will consider in turn what each of these heuristics says about the objective nature of the Choice.

3.2 Wright's Heuristic Tests

3.2.1 Width of Cosmological Role

According to Wright, a discourse can be considered to have a wide cosmological role "just in case mention of the states of affairs of which it consists can feature in at least some kinds of explanation on contingencies which are not of that sort – explanations whose possibility is not guaranteed merely by the minimal truth aptitude of the associated discourse" (Wright, 1992, 198). Explanations which can feature in many diverse discourses and which are more than trivially informative are ones that are not narrowly bound to their initial subject matter. The idea here is that the discourse in question seems more apt for a realist construal if it figures in the

explanations of many different discourses because they appear to be ‘self-standing’ – the explanations are not inseparably bound up with the discourse in question.

As an example, compare the discourses regarding a particular physical property, say roughness, and a particular moral property. In the case of roughness, providing a substantial explanation for why a piece of sandpaper is rough can be used in more discourses than simply that one. It can be used to explain the perception of roughness against one’s skin, the ideas of differing mechanical friction between sandpaper and silk, notions of texture and variability of surface appearance, and others. The explanation of roughness seems to be based on something self-standing that can be engaged in a variety of arenas including perception, mechanics, and aesthetics. This explanation would seem more apt for a realist construal. Wright contrasts this with a explanation of a particular moral property, like goodness. In what other discourses do explanations of moral goodness have substantial and independent relevance besides that of moral discourse? Wright argues that the difficulty of locating other arenas suggests that goodness may be fundamentally and inseparably tied up with moral discourse and therefore not a self-standing property.⁴

So what about the Choice? In studying the objectivity of logic itself, Shapiro (2000) claims that logic has the widest of all possible cosmological roles. He argues that since logic is critically involved in explanations in any disciplined discourse, this is very strong evidence for a metaphysical realism and objective understanding of logical concepts.

⁴Although there could be debate as to whether ‘goodness’ figures in a variety of discourses, the example is meant simply to be illustrative of how cosmological role *could* have comparatively variable width for different properties.

We explain the tides and the orbits of the moon by *deducing* the relevant data from gravitational theory (and initial conditions). We explain the divergence of species by deduction from principles of biology (and specific hypothesis about mutations, populations splittings, etc). We explain trends in the stock market, global warming, etc. in like manner. It is no accident that logic is said to be completely general. On the official definition of cosmological role, nothing is wider than logic" (Shapiro, 2000, 361).

Although a logic does not provide any explanation in and of itself directly, no explanation of the sort identified by Shapiro would be possible without employing said logic. It must be noted, though, that Shapiro is speaking of logical explanation entirely on and completely at the object level. But, as clarified earlier, the Choice resides instead at the meta-logical level. Once the Choice has been made, the resulting logic is general and figures widely in nearly all explanations.

Any explanation *about* the Choice, rather than having the wide genericity of a fixed object logic, seems instead to have extremely limited cosmological role. The features and properties that will figure in any explanation about the Choice must be exclusively meta-logical in nature. In what other discourses do meta-logical explanations often provide useful, meaningful, or substantive information? Explanations of the kind identified by Shapiro employ none of the meta-logical apparatuses used in a discussion of the Choice. These concepts, such as soundness, completeness, and compactness, are employed almost exclusively to make sense of the narrow space of comparing rival logics. These ideas do not feature in explanations of a non-logical nature. The explanations found in the discourse regarding the Choice seem to lack a strongly 'self-standing' property and are intimately tied to meta-logical discussion alone. This analysis fails to make strong conclusive claims regarding the objectivity

of the Choice, but it does seem to bleed significant heuristic support for the Choice to be one apt for a realist construal with objective justification.

3.2.2 Cognitive Command

Cognitive Command is a test that tries to determine the external representational nature of a discourse. An accessible route to understanding this test is through analogy. Wright reasons that it is a priori true that devices, like a camera or Xerox machine whose entire function is to produce an accurate representation of an external subject, can only diverge in their product significantly when they are broken. Two such devices which are designed to produce accurate representations of an external reality can only diverge due to malfunction. By analogy, it must be a priori true when dealing with a discourse where beliefs represent external states of affairs, that two human beliefs or opinions can only diverge when at least one is in error. By taking the contrapositive, an interesting heuristic dimension of capability for metaphysical realism and objectivity emerges. If two thinkers can hold beliefs which diverge on truth values in a discourse and not a priori conclude that one must have made a cognitive error, then the discourse does not have strong evidence for adopting a realist construal.

So again, what of the Choice? If thinker *A* deems that constructivist logic is the correct one and thinker *B* deems that second-order logic is the correct one, what can we say a priori about the presence of cognitive errors? The two thinkers will exhibit status and extension disputes for particular classes of sentences, one asserting their truth and the other demurring. In this case, we could offer the Law

of Excluded Middle or single sentence Law of Induction as examples. Here we have particularly illustrative examples of a status dispute (with excluded middle) and an extension dispute (with induction) as introduced in §1. Within each thinker's object logic, both can construct valid arguments for why the other has committed grave inferential errors. Both then have object level justifications for the cognitive shortcomings of the other. It also seems on first glance that this shortcoming can be understood as an a priori truth. However, to correctly answer the question posed by the Choice, the assertion of a *correct* logic requires something stronger. Justification at the meta-logical level is needed. Is any argument for one logician's meta-logical error knowable a priori? To answer in the affirmative seems to directly beg the question posed by the Choice. The investigations of this paper attempt to show that answering the Choice is no simple task and that presently formulated attempts fall short in one capacity or another. Answering the Choice successfully would provide definitive answer to the dimension of Cognitive Command for the discourse of logic choice, but the reversed heuristic considerations seem to provide no more particularly substantial information. Rather than lending support to one answer or another, considerations of Cognitive Command seem to simply beg our question.

3.2.3 Euthyphro Contrast

For any discourse which is epistemically constrained, the concepts of truth and best opinion will agree in extension by definition. What can be asked, however, is what is the direction of fit: does the best opinion coincide because it tracks the truth, or is the truth constituted by the determination of a best opinion? Wright's study of such

chicken and egg arguments focuses on the debate between Socrates and Euthyphro in Plato's *Euthyphro* about the direction of fit for piety and the opinions of the gods. In analyzing this situation, Wright develops a highly structured test for determining when a given discourse can manage a substantively subjective direction of fit or a purely objective one.

The developed test centers around Wright's provisional equations, presented generically as: under ideal conditions C , ideal agent A determines x to be true if and only if x is true. If such an equation can be found to be true under certain constraints, then it is reasonable to conclude that an agent to world direction of fit, with essential subjectivities, can succeed for the discourse in question. These constraints are that (1) the C conditions must be substantial and non-trivial, (2) that they must be logically independent of the consequent biconditional, and (3) that the provisional equation must be not only true, but true a priori. These provisional equations seem informative with respect to many discourses where questions about objectivity are encountered, but seem to fail miserably when trying to inform on the Choice.

The provisional equations suggested by the Euthyphro contrast are fundamentally argumentative in nature. They are constructed as implications on biconditions in order to persuade and secure certain conclusions. Immediately, this seems to presuppose a great deal of logical framework as true and available for use. If the direction of fit in question regards logical notions themselves, on what grounds can we claim to be able to fully construct the provisional equations? For Wright, the provisional equations are *not* intended to be strictly metalogical in nature, but

instead are meant to be statements to be evaluated within the discourse of interest. However, it is not directly obvious that the logical framework needed to construct such statements is fully available to us meta-logically without presupposing an effective answer to the Choice.

Additionally, at least two of the constraints on the provisional equations are also essentially Choice-dependent in nature. Condition (2) clearly depends on fixed interpretations of logical necessity. Condition (3) focuses on the status of certain sentences being a priori true or not. These conditions shed serious negative light on the ability to use the Euthyphro Contrast to meaningfully show that the Choice is objective. Too many working parts of the test are bound up with the question itself. Even if the provisional equations were deemed acceptable despite these complications, it seems an open question whether or not this test adjudicates a dispute between two rival logicians.

3.3 Conclusions from Wright's 'Standard' Account

Having found little support for the objectivity of logic choice on Wright's heuristic grounds, the options seem to either accept that logic choice may have to admit non-objective construals or attempt a direct construction of an objective epistemic reason (a justification) for a particular set of logical principles. Due to the philosophically unpalatable nature of the former route, we will now consider an attempt of the latter.

4 Rule-Circular Justification

In an attempt to provide a justification for deductive reasoning, Boghossian (2001) argues that certain forms of rule-circular arguments may be admissible in discourses similar to and including the subject of logic. Boghossian's argument focuses mainly on justifying the use of *modus ponens* as a rule of inference, but the structure of his argument has the potential to be extended into other debates regarding the justification of other deductive logical concepts. After a brief recapitulation of Boghossian's argument, we will explore how these thoughts can be used to defend the acceptance of multiple nonequivalent logics simultaneously – known henceforth as *logical pluralism*. Whether or not this pluralism casts doubt on the possibility of objective logic choice will be investigated.

4.1 Justification for Modus Ponens

In attempting to provide a solid footing for (MP), Boghossian starts by asking what objective epistemic reasons, serving as justifications, must look like. First, he argues that any posited rationale must be epistemically accessible. Any rationale regarding knowledge which is not knowable is ultimately unhelpful in any discussion of epistemic foundations. Additionally, he suggests that there might even be something like a contradiction in terms in suggesting that there are unknowable epistemic truths, as noted earlier in the paper. With this move, Boghossian constrains the discussion. For the purposes of establishing his argument and developing its analogs with the Choice, we will grant this constraint.

Boghossian moves from here to the problem that if an objective epistemic reason

can be known, then it should be justifiable, and the search moves into the realm of justification. In parsing out the field of possible solutions, Boghossian notes that a justification for deductive inference must either be inferential or non-inferential. The two main contenders on the non-inferential side are intuitionism and default-reasonableness. Intuitionism asserts that humans possess a quasi-perceptual faculty, which when exercised gives us the ability to access and become aware of necessary truths. This faculty is often understood as “rational intuition.” Default-reasonability, on the other hand, is the assertion that certain beliefs are reasonable in and of themselves, without justification from evidence, argument, or even intuition. Both of these solutions are quickly dispatched as unsatisfactory: the first requires too many extra perceptual and metaphysical entities; the second seemingly states the conclusion without providing the desired explanation or justification. On these grounds, Boghossian suggests that an inferential argument may be the only route to a successful justification of an objective epistemic rationale for deductive inference.

Centrally, Boghossian claims that any successful inferential justification of (MP) will ultimately need to evoke that rule at least once, leading to a kind of circularity in argument. This circularity is due to the fact that in order to properly reason about any object logic on a meta-logical level, thinkers need at least a basic set of argumentative tools in order to have productive meta-logical discussion. This thought is crucial to both Boghossian’s local argument regarding (MP) and any larger discussion regarding logics in general because it shows that certain notions involved in the study of logic invariably use logical apparatus; circularity of one form or another seems to be an inevitable consequence of meta-logical argumenta-

tion. Note that this reinforces the results obtained from the earlier considerations of Wright's tests for objectivity.

Boghossian thereby claims that the use of 'rule-circular justification' would be needed in order to salvage objective epistemic reasons, and he argues for its legitimacy. A rule-circular justification is when an inference rule is used *inside* an argument to justify the legitimacy of that very inference rule. Boghossian argues that this kind of circularity is legitimate because it successfully avoids the traditional pitfalls of what he labels gross-circularity: 'bad-company' and begging the question.

Bad company is the effect that a systematic inconsistency allows the proof of any arbitrary proposition. Gross circularities invite bad company by allowing a valid argument for any arbitrary conclusion.⁵ Additionally, gross-circularities often beg their question by taking as a premise the intended conclusion, thereby ensuring the result trivially. Boghossian analyzes the function of rule-circularity in the proposed justification for (MP) and concludes that neither of these difficulties is encountered.⁶ Although rule-circular justification seems to gain some traction in Boghossian's analysis, it starts to come apart when applied to other meta-logical questions.

When extended to considerations regarding the Choice, rule-circular argumentation seems at first to support the possibility of objective epistemic reasons for the Choice. For Boghossian, (MP) is objectively justified through the use of a rule-circular argument which guarantees the truth of its consequence; that is, by

⁵For an example of bad company, assume a simple propositional logic as our working model. Additionally, say I have shown that both A and $\neg A$ are true. In doing so, I can now show any proposition B to be true through the use of the tautology $A \rightarrow (\neg A \rightarrow B)$. The combination of these three premises allows for a valid proof of B . Since B is arbitrary, we have our bad company. This is the reverse of a reductio ad absurdum.

⁶For a detailed analysis how rule-circular arguments avoid these pitfalls, see Boghossian (2000) pages 51-55

using (MP), it is possible to construct an argument that shows how (MP) is a truth-preserving and consistent logical operator. This argument can be used to support the meaning-constitution arguments for inference in many rival logics. By replacing (MP) by particular inference rules as defined in each rival logic, objective warrant for the use of these inference rules is established. This would suggest that justifications for the logical tautologies could be held with a stronger justification than those offered along non-inferential or relativist routes. However, this optimism is severely dampened by the recognition that the argument can apply equally well to the choice of many logics. This admits a form of logical pluralism that can be seen as undercutting notions of objectivity in the discourse. To simplify the discussion of this point, we will limit ourselves to considering constructivist and first-order logics as the only candidates for logic choice.

Boghossian's requirements for a rule-circular justification are satisfied by both constructivist and first-order construals of the logical connectives. In both cases, the meaning-constitution requirement is fulfilled. Constructivist logical *AND* as well as first-order logical *AND* (henceforth \wedge_{con} and \wedge_{first} respectively), as well as the negations \neg_{con} and \neg_{first} , can be used to construct the useable logical syntaxes and semantic of inferences which are interpretable and meaningful.⁷ There is no confusion among practitioners regarding what is meant by these connectives nor what the semantic character of compound sentences using these connectives amounts to. Boghossian also requires that a belief not be epistemically irresponsible; a belief is epistemically irresponsible in "the absence of a reflectively appreciable

⁷We define $A \Rightarrow_{con} B \equiv \neg_{con} ((\neg_{con} A) \wedge_{con} B)$, $A \Rightarrow_{first} B \equiv \neg_{first} ((\neg_{first} A) \wedge_{first} B)$. Additionally, we allow the semantic truth value of the left hand side of each equivalence to assume the same value as the right hand side.

warrant for the belief (which can sometimes assume the form of the presence of a reflectively appreciable warrant for its negation)” (Boghossian, 2001, 49). Since both systems are provably known to be consistent and sound, the requirement of epistemic responsibility is guaranteed. Using the structure of Boghossian’s argument, then, both of these logical interpretations would be objectively justified, in a rule-circular way.

However, admitting this justification of both constructivist and first-order logic leads to some problematic results. The first worry concerns one’s ability to interpret a given sentence partially under one logic choice and partially under another. These languages are strictly formal, not arbitrary; there are strict rules regarding the formation and interpretation of well-formed formulas, unlike many natural languages. The immediate unintelligibility of what could be meant by the sentence $P \wedge_{con} Q \wedge_{first} R$ poses serious problems for using these interpretations simultaneously. Since it is joined with a constructive connective, is Q allowed to be any first-order well-formed formula or must it be limited to a constructivist formula? Does conjoining a first-order sentence with a constructive one limit the range of the formula to constructive truth-values, extend the constructive ones, or simply lack meaningful truth-value entirely? These considerations seem to show that these interpretations can only be successfully employed in a purely disjoint manner. The result of this is a bifurcation of the class of logical truths into two disjoint sub-classes, the constructive truths and the classical truths.

Allowing this logical bifurcation raises serious questions as to the objective nature of the discourse, an objective nature that Boghossian’s argument structure

was supposed to justify and defend. The use of rule-circularity was claimed to offer a middle ground between strong independent warrant (seemingly impossible for fundamental logical concepts) and widespread epistemic relativism. Instead, we are led by this argument to a form of logical pluralism regarding admissible logics. Should this worry us? That the following example will show that it should.

Boghossian admits that his construction ultimately amounts to something that can at its strongest only be non-suasive. It can explain and justify a believer's position, but it fails to be persuasive to convert or convince any sufficiently hard-nosed skeptic. The fact that his argument also supports a form of logical pluralism seems to show, though, that it may be weaker than being non-suasive. Rule-circular justification seems to fail to provide the suitable defense of objectivity we have been desiring. The normative character we fixed earlier is one that allowed for the recognition of mistakes: someone looking over another's shoulder could point out an inferential error, argue for its falsity, and correct the mistaken person without resulting in any personal relativism. The non-suasive nature of Boghossian's objective epistemic reasons seems to lose this character. We will now explore this in greater detail.

4.2 Normativity and Objectivity in a Logical Pluralism

In order to highlight this slippage, let's consider some of the differences between the rival logics under consideration. Between the logical interpretations, there are sentences that gain a certain truth value in first-order logic that are not obtained in constructivist logic. The most notable of these is the law of double negation,

$\neg(\neg(A)) \Leftrightarrow A$. Under first-order interpretation, this sentence is always true. Under a constructivist interpretation, this is not the case. For a constructivist logician reviewing an argument over a friend's shoulder, the use of double negation would be in error. According to Boghossian's argument, the constructivist would have objective warrant for claiming that the statement is not admissible. However, as we have seen, the first-order logician *also* has legitimate objective warrant for claiming that the double negation is admissible and fully justified. The question now becomes apparent – which logician is correct? Now the problem of logical pluralism comes to a point: if we accept Boghossian's rule-circular arguments, our notions of objective justification lose most of their broad normative character. No longer can we say a particular statement is logically true or false; we must qualify it as *constructively* or *classically* true or false. Allowing rule-circularity was an attempt to move away from epistemic relativism, but it seems, to an appreciable extent, to have put us back in its grip.

From this point, we can now consider whether or not Boghossian's rule-circular solution meets our desired criteria for the Choice. In allowing for objective justification of multiple logics, have we also maintained satisfaction of the fixed normative condition or has it slipped away? The normative condition we have fixed is a weaker condition than that imposed by Wright's Cognitive Command; rather than requiring that a cognitive error be identifiable only a priori, our condition simply requires that cognitive errors be simply recognizable.

However, by requiring an ability for 'over-the-shoulder' checks, the condition seems satisfiable in solutions that permit only a monadic truth predicate. To see

why, again consider the two logicians above. In order for the constructivist logician to have grounds for the normative claim that the first-order logician had made a logical *mistake*, she must demonstrate at what point false conclusions supposedly followed from true premises. One example could be the use of the Law of the Excluded Middle. Note, the normative claim would not be that her logician friend had made a *constructivist mistake*, but simply a *mistake*. The normative condition fixed at the beginning of this investigation is one that can only take a binary status: correct or mistaken. The strict inability of our normative considerations to have plural description imposes the condition that our truth predicate also be monadic. A given argument is either correct or mistaken; its conjunctive sentence is either true or false.

The objective solution obtained through Boghossian's line of argument however permits a multivariate truth predicate. A given argument can be valid within justifiable logic *A* while not valid in justifiable logic *B*, vice-versa, or both. The conditions for a logical pluralism require minimally the need for a two-place truth predicate: the first argument for the predicate being a (potentially empty) set of sentences to be evaluated, the second argument denoting the logical consequence relation under which truth is predicated. Satisfaction of the objective condition through rule-circularity has led to a violation of our strong normative condition. The unusual front door solution to the objective needs of the problem has admitted an attack through the back door on the normative.

So what can be done to prevent this unfortunate side-effect on the normative dimension? One route is offered by Greg Rettall and JC Beall's (2006): relax the

normative condition. Restall and Beall argue for a broadly pluralist view towards logical consequence. Realizing the normative quagmire surrounding a pluralist view, they suggest the following normative criterion:

“What we have here is a case of a necessary evil, or an epistemic dilemma. Epistemic criteria need not always agree, any more than moral criteria....In particular, we hold that it is a mistake to assert the premises of a valid argument while denying the conclusion. This norm, however, may be trumped, and we may have reason to violate it” (Beall and Restall, 2006, 17-18).

However, what would count as ‘good reason’ is left unspecified and unformalized. Clearly, these conditions would necessarily be of non- or extra-logical nature, potentially including the notions we noted at the beginning, of simplicity, clarity, completeness, or finitism.

This weakening of the normative condition entailed by logical pluralism seems to go too far. The normative condition was placed in order to allow for the correction of wayward argumentation based on clearly understood, rigorously expressed, and fully verifiable conditions. These requirements are used to ward off a persistent epistemic relativism. The solution offered by Restall and Beall, by allowing normative trumping for “good reason,” seems to put the solution back to square one, unable to cope with a relativist about what would constitute such reasons.

Now it is also possible to see why the pragmatic suggestion of just using ubiquitous classical logic is unsatisfactory. Pragmatism alone is not one of our four desired criteria. Additionally, it should be noted that a Choice exists at the meta-logical level as well. More meta-logics than just classical have been developed, including the semantic anti-realist constructivism pursued by Michael Dummett

(1998), as well as Carnap's *Language I* (1937), which is its own meta-logic. Pragmatic considerations fail to preclude that either of these meta-logics is not the *correct* one outright. Additionally, any claim of downward justification from the meta-logic to justify the object logic must still provide a cogent story for the satisfaction of our other four desired criteria. Accepting Boghossian's rule-circular justification is highly problematic in this situation because it opens the door for a logical pluralism at all logical levels. A classical interpretation pushed downward still leaves an object language that can be meaningfully interpreted classically or constructively. The rule-circular story allows for an upward justification of both of these logics, but undermines the original insistence on classical logic with multiple justified logics. On the other hand, the failures of Wright's account to be informative on logical notions persist on both the object- and meta-logical level for a Choice of classical logic.

5 Conclusion

With many differing logics available to the modern philosopher and mathematician alike, the choice between them motivates a centrally important foundational question, the Choice: which logic is the correct one? In order to answer this question, we sought a story for the justification of the Choice that met four substantial conditions. The story should be one which maintains consistency, should be robustly epistemically accessible, should allow for normative evaluation for correct argumentation, and should allow for objective warrant devoid of essential subjectivities. Two potential stories have been considered in this work: Wright's traditional realist one as

well as Boghossian's rule-circular one.

Wright's realist story of objectivity with regards to the Choice was not substantially informative on the question. Rather than providing a strong positive or negative answer to the question, Wright's tests instead left the subjective and objective camps in a conceptual stalemate. Arguably, much of the failure for Wright's story to inform on the Choice is due to its sophisticated structure and reliance on tests developed *within* a framework of developed logical consequence. The tests thereby would presuppose an answer to the Choice rather than helping to elucidate it.

Boghossian's rule-circular arguments seemed to bring us closer to a definitive answer on the Choice. Its direct engagement with questions surrounding logical implication and consequence allowed for evaluation of the most critical aspects of the Choice. However, Boghossian's constructions admit a troubling logical pluralism that seems to fail the strong normative criterion.

Three mutually exclusive conclusions seem to come from the considerations stemming from the pluralism result.

1. Boghossian's rule-circular method may simply be largely bankrupt for answering the Choice.
2. The normative condition we have assumed is simply too strong for any successful story.
Or,
3. The four conditions for a successful story might be deeply conjugate, mutually overdetermining the possible answers to the Choice to the null set.

Of the three possibilities, the first seems the most likely but the last seems the most interesting question for further investigation. Although thinkers like Beall

and Rustall have opted for the middle ground found in second possibility, serious consideration should be given to all the options.

So, have we just ended up where we started, having gone in a large circle? Although we have started and ended with the Choice still open, a considerable narrowing of what could nontrivially be an answer to the Choice has been determined. There are certainly two distinct ways of interpreting these results for the question posed as the Choice. One would be to conclude that the question is not well-formed. However, rather than directly answering the Choice, this conclusion merely deflects the investigation raising deep questions as to the requirements for well-formed questions about logic. Another would be to conclude that we have elucidated a deep and delicate relationship between our dimensions of objectivity and normativity that persist throughout logical discussion; one, or both, may be too strong, too vague, or inappropriate for the discussion. A more promising continuation of this study would be to see if robust conclusions can be formed regarding the structural interrelationship between consistency, accessibility, normativity, and objectivity on the foundations of logic.

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